A GAME-THEORETIC COOPERATIVE ADVERTISING MODEL: THE FEASIBILITY OF THE DISTRIBUTOR'S INVOLVEMENT IN A MANUFACTURER-DISTRIBUTOR-RETAILER CHANNEL

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Abstract: Most cooperative advertising works to date consists of only two parties: the manufacturer(s) and the retailer(s). This work uses Game theory to address the possibility of the distributor being an integral part of cooperative advertising supply chain. It thus considers a manufacturer-distributor-retailer supply channel in which the manufacturer is the Stackelberg leader, while the distributor and the retailer are the first and second followers, respectively. The players' strategies are their prices, advertising efforts and advertising subsidy. The work uses price and demand multiplicative effect to model consumer demand, and obtains the players' equilibrium prices, advertising strategies and payoffs. It shows that with noninvolvement of any of the players either directly or indirectly in advertising, his payoff becomes very large at the expense of the other channel members. Particularly it shows that the distributor must be either directly or indirectly involved in advertising for him to be an integral part of the supply chain.

Keywords: Cooperative advertising, demand-price function, Game theory, supply chain

Introduction

Cooperative advertising is an advertising method in which the manufacturer pays for all or a fraction of the retail advertising expenditure incurred by the retailer in the process of advertising the product. This method is usually used by the manufacturer to encourage the retailer to advertise the manufacturer's product. The fact that the retailer is much closer to the consumers compared to the manufacturer serves as an incentive for the subsidy. In addition, the retailer has a better understanding of the local setting, and can engage in advertising using local media at lower cost.

In the traditional cooperative advertising models only the manufacturer and retailer are considered as members of the supply chain. But a critical examination suggests that there is the need to incorporate a third party. This is because a lot of manufacturers do not deal directly with their retailers. The distributors usually stand as a link between them. In this work we will incorporate the distributor into the classical manufacturer-retailer cooperative advertising model to deal with a manufacturer-distributor-retailer supply chain, and consider the viability of such a channel.

A lot of works in the advertising literature examines supply chain relationship using static models (Dant and Berger, 1996; Kim and Staelin, 1999; Karray and Zaccour, 2006, 2007). Such approach helps to explore interactions among various factors associated with cooperative advertising. Huang and Li (2001), Huang *et al.* (2002) and Li *et al.* (2002) observed that there exists a significant difference between the manufacturer and retailer's advertising expenditure, and studied advertising decisions by the channel members for different kinds of relationships. Yue *et al.* (2006) studied channel coordination in a manufacturer-retailer cooperative advertising relationship when the manufacturer gives price discount to the retailer. This is an extension of Huang and Li (2001). Another extension of Huang and Li (2001) was done by Xie and Neyret (2009). They studied optimal pricing decisions and cooperative advertising by using four forms of manufacturer-retailer relationships. Xie and Wei (2009) addressed channel coordination. They obtained optimal cooperative advertising strategies and equilibrium prices in a distribution channel. Static cooperative advertising in a manufacturer-retailer channel on fashion and textile was considered by He *et al.* (2014). They introduced two approaches in subsidizing advertising as a means of coordinating the channel.

This work considers cooperative advertising in a manufacturer-distributor-retailer channel with the manufacturer as the Stackelberg leader, the distributor as the first follower and the retailer as the second follower. It will consider the three channel members to be involved in a hierarchical game where the retailer is directly involved in advertising the manufacturer's product, and the manufacturer is indirectly involved by participating in retail advertising. The distributor is neither directly nor indirectly involved in advertising, but decides his price to the retailer just as the manufacturer and retailer decide their prices to the distributor and consumer, respectively.

Based on this setting, the work will answer questions on the optimal retail advertising effort; the manufacturer's participation rate; the players' optimal prices; the players' payoffs, and from these consider the feasibility of the distributor being a part of a cooperative advertising supply chain.

Basic Market Structure

This work deals with a channel involving a manufacturer, a distributor and a retailer. The manufacturer supplies the distributor with the goods who then supplies the retailer. The retailer in turn supplies the consumer. They both sell only the manufacturer's brand among substitutes. The retailer's decision variables are his advertising expenditure A and unit price P_R at which the goods is sold to the consumer. The distributor decides on his price P_D which he sells to the retailer, while the manufacturer's decision variables are his wholesale price P_M to the distributor and participation rate β. The participation rate is also known as the subsidy rate. It is the percentage of advertising expenditure which the manufacturer gives the retailer to subsidise the cost of advertising. The demand function D depends on the advertising level A and retail price P_R. Thus we have that

$D(P_R, A) = f(P_R)g(A)$, (1)

Where f indicates the impact of price P_R on the demand D and g represents the impact of retail advertising on D.

The use of multiplicative effect of price and advertising in modeling demand is a common feature in the literature (Kuehn, 1962; Thompson and Teng, 1984; Jorgensen and Zaccour, 1999; Yue *et al.*, 2006; Xie and Wei, 2009). We will assume that f is linearly decreasing with respect to P_R. This demand function is well known in the literature (Jeuland and Shugan, 1988; Weng, 1995). It is given by

$f1 - (P_R) = \theta P_R$ (2)

where $\theta > 0$.

Now, to aid the simplification of our expressions, we let $0 < f \leq 1$, thus normalizing f to 1. The function g is given by

$g(A) = \alpha\sqrt{A}$, (3)

where α represents the retail advertising effectiveness which indicates the impact of retail advertising on sale. The function g is an increasing function of A. This is in accordance with the usually observed advertising saturation effect – a situation where any additional expenditure made on advertising leads to diminishing returns.

Now, from (2) and (3) we have that (1) becomes

$$D(P_R, A) = (1 - \theta P_R)\, \alpha\sqrt{A}. \quad (4)$$

Thus the payoffs (profits) of the retailer, distributor and manufacturer are

$$R_{Prf} = (P_R - P_D)(1 - \theta P_R)\alpha\sqrt{A} - (1 - \beta)A, \quad (5)$$

$$D_{Prf} = (P_D - P_M)(1 - \theta P_R)\alpha\sqrt{A} \quad (6)$$

and

$$M_{Prf} = P_M(1 - \theta P_R)\alpha\sqrt{A} - \beta A \quad (7)$$

respectively.

The players' decision sequence

We will model the players' decisions as a sequence of non-cooperative game. The manufacturer will be considered as the Stackelberg leader, the distributor as the first follower, and the retailer as the second follower. The Stackelberg equilibrium will be determined using backward induction. First, the manufacturer who is the leader of the game decides his price P_M and participation rate β. Next, based on these decisions the distributor makes his price P_D known to the retailer. In reaction to these decisions the retailer decides his retail price P_R and advertising effort A. Thus given the manufacturer and distributors' decisions, the retailer's objective is to

$$\max\ R_{Prf} = (P_R - P_D)(1 - \theta P_R)\alpha\sqrt{A} - (1 - \beta)A \quad \text{s.t}\quad A \geq 0, P_R \geq 0. \quad (8)$$

To obtain the value of P_D the distributor maximizes the problem

$$\max\ D_{Prf} = (P_D - P_M)(1 - \theta P_R)\alpha\sqrt{A} \quad \text{s.t}\ P_D \geq 0. \quad (9)$$

Similarly the manufacturer's price P_M and the optimal participation rate β are obtained by maximizing

$$\max\ M_{Prf} = P_M(1 - \theta P_R)\alpha\sqrt{A} - \beta A \quad \text{s.t}\ P_M \geq 0,\ 0 \leq \beta \leq 1. \quad (10)$$

From the above discussion we have the following results:

The players' prices, advertising and subsidy strategies

Proposition 1: In a manufacturer-distributor-retailer supply chain the retail advertising effort is given by

$$A = \frac{\alpha^2(1 - \theta P_R)^2(P_R - P_D)^2}{2^2(1 - \beta)^2}, \quad (11)$$

the pricing decisions are given by

$$P_R = \frac{1 + \theta P_D}{2\theta}, \quad (12)$$

$$P_D = \frac{1 + 3\theta P_M}{4\theta}, \quad (13)$$

$$P_M = \frac{4 - \beta}{16\theta - 13\theta\beta}, \quad (14)$$

and the subsidy rate is given by

$$\beta = \begin{cases} \dfrac{16\theta P_M - 3(1 - \theta P_M)}{16\theta P_M + 3(1 - \theta P_M)}, \\ 0, \quad \text{otherwise}. \end{cases} \quad (15)$$

Proof: Maximizing (8) with respect to A we have

$$\frac{\partial R_{Prf}}{\partial A} = (1 - \theta P_R)(P_R - P_D)\alpha\frac{1}{2\sqrt{A}} - 1 + \beta = 0.$$

Rearranging we have (11).

Putting (11) into (8) we have

$$\max\ R_{Prf} = \frac{\alpha^2(1 - \theta P_R)^2(P_R - P_D)^2}{2^2(1 - \beta)} \quad (16)$$

$$\text{s.t}\ P_R \geq 0.$$

Maximizing (16) with respect to P_R we have

$$\frac{\partial R_{Prf}}{\partial P_R} = \frac{\alpha^2}{4(1 - \beta)}[(1 - \theta P_R)^2\big(2(P_R - P_D)\big) + (P_R - P_D)^2(2\big((1 - \theta P_R)\big)(-\theta) = 0$$

$$\Rightarrow \quad 1 - \theta P_R = (P_R - P_D)\theta,$$

which leads to (12).

Using (11) and (12) in (9) we have

$$\max\ D_{Prf} = \left(\frac{1 - \theta P_D}{2}\right)^3 \frac{\alpha^2(P_D - P_M)}{2\theta(1 - \beta)}. \quad (17)$$

$$\text{s.t}\ P_D \geq 0.$$

Maximizing (17) with respect to P_D we have

$$\frac{\partial D_{Prf}}{\partial P_D} = (P_D - P_M)(3)\left(\frac{1 - \theta P_D}{2}\right)^2\left(-\frac{\theta}{2}\right) + \left(\frac{1 - \theta P_D}{2}\right)^3 = 0$$

which leads to (13).

Using (11) and (12) in (10) we have

$$\max\ M_{Prf} = \frac{\alpha^2 P_M}{2\theta(1 - \beta)}\left(\frac{3 - 3\theta P_M}{8}\right)^3 - \beta\left(\frac{\alpha}{2\theta(1 - \beta)}\right)^2\left(\frac{3 - 3\theta P_M}{8}\right)^4. \quad (18)$$

$$\text{s.t}\ P_M \geq 0, \quad 0 \leq \beta \leq 1.$$

Maximizing (18) with respect to P_M we have

$$\frac{\partial M_{Prf}}{\partial P_M} = 3 - 3\theta P_M - 9\theta P_M + \frac{12\theta\beta(3 - 3\theta P_M)}{16(1 - \beta)\theta} = 0.$$

Rearranging we have (14).

Further maximizing (18) with respect to β we have

$$\frac{\alpha^2(3 - 3\theta)^3 P_M}{2(8^3)(1 - \beta)^2} - \frac{\alpha(3 - 3\theta P_M)^4}{2^2(8^4)\theta}\left[\frac{(1 - \beta)^2 + 2\beta(1 - \beta)}{(1 - \beta)^4}\right] = 0$$

$$\Rightarrow \quad \beta = \begin{cases} \dfrac{16 P_M\theta - 3(1 - \theta P_M)}{16 P_M\theta + 3(1 - \theta P_M)}, & 16 P_M\theta > 3(1 - \theta P_M) \\ 0 & \text{otherwise} \end{cases}. \quad \blacksquare \quad (14)$$

Unsubsidised equilibrium strategies and payoffs

Proposition 2: When retail advertising is not subsidised, the retailer, distributor and manufacturer's pricing decisions are

$$P_R = \frac{23}{32\theta}, \qquad (19)$$

$$P_D = \frac{7}{16\theta} \qquad (20)$$

and

$$P_M = \frac{1}{4\theta} \qquad (21)$$, respectively.

The advertising effort is

$$A_{(\beta=0)} = \left(\frac{9}{32}\right)^4 \left(\frac{\alpha}{2\theta}\right)^2, \qquad (22)$$

and the payoffs are

$$R_{Prf=0} = \frac{6561}{4194304}\left(\frac{\alpha}{\beta}\right)^2, \qquad (23)$$

$$D_{Prf=0} = \frac{2187}{1048576}\left(\frac{\alpha}{\beta}\right)^2 \qquad (24)$$

and

$$M_{Prf=0} = \frac{729}{262144}\left(\frac{\alpha}{\beta}\right)^2. \qquad (25)$$

Proof: Since there is no subsidy from the manufacturer we have that $\beta = 0$. As such (14) becomes

$$P_M = \frac{4}{16\theta} = \frac{1}{4\theta}.$$

Equation (13) becomes

$$P_D = \frac{1 + 3\theta\frac{1}{4\theta}}{4\theta} = \frac{7}{16\theta},$$

and (12) becomes

$$P_R = \frac{1 + \theta\frac{7}{16\theta}}{2\theta} = \frac{23}{32\theta}.$$

Using (19) and (20) in (11) we have

$$A_{(\beta=0)} = \frac{\alpha^2\left(1 - \theta\frac{23}{32\theta}\right)^2\left(\frac{23}{32\theta} - \frac{7}{16\theta}\right)^2}{2^2}$$

$$= \left(\frac{9}{32}\right)^4 \left(\frac{\alpha}{2\theta}\right)^2.$$

Using (19), (20) and (22) in (5) we have

$$R_{Prf} = \left(\frac{23}{32\theta} - \frac{7}{16\theta}\right)\left(1 - \theta\frac{23}{32\theta}\right)\alpha\left(\frac{\alpha}{2\theta}\right)\left(\frac{9}{32}\right)^2 - \left(\frac{9}{32}\right)^4\left(\frac{\alpha}{2\theta}\right)^2$$

$$= \frac{6561}{4194304}\left(\frac{\alpha}{\beta}\right)^2.$$

Using (19), (20), (21) and (22) in (6) we have

$$D_{Prf} = \left(1 - \theta\frac{23}{32\theta}\right)\left(\frac{7}{16\theta} - \frac{1}{4\theta}\right)\alpha\left(\frac{\alpha}{2\theta}\right)\left(\frac{9}{32}\right)^2$$

$$= \frac{2187}{1048576}\left(\frac{\alpha}{\beta}\right)^2.$$

Using (19), (21) and (22) in (7) we have

$$M_{Prf} = \left(1 - \theta\frac{23}{32\theta}\right)\frac{1}{4\theta}\alpha\left(\frac{\alpha}{2\theta}\right)\left(\frac{9}{32}\right)^2$$

$$= \frac{729}{262144}\left(\frac{\alpha}{\beta}\right)^2. \quad \blacksquare$$

Subsidised equilibrium strategies and payoffs

Proposition 3: Suppose the manufacturer subsidizes the retail advertising effort, then the retailer, distributor and manufacturer's pricing decisions are

$$P_R = \frac{10}{13\theta}, \qquad (26)$$

$$P_D = \frac{7}{13\theta} \qquad (27)$$

and

$$P_M = \frac{5}{13\theta} \qquad (28)$$

respectively.

The advertising effort is given by

$$A_{(\beta>0)} = \left(\frac{3\alpha}{52\theta}\right)^2, \qquad (29)$$

and the payoffs are

$$R_{Prf>0} = \left(\frac{3}{26}\right)^3\left(\frac{\alpha}{\beta}\right)^2, \qquad (30)$$

$$D_{Prf>0} = \frac{9}{4394}\left(\frac{\alpha}{\beta}\right)^2 \qquad (31)$$

and

$$M_{Prf>0} = \frac{63}{17576}\left(\frac{\alpha}{\beta}\right)^2 \qquad (32)$$, respectively.

Proof: Since the manufacturer subsidises the retail advertising effort it follows that by using (15) in (14) we have

$$P_M = \frac{4 - \frac{16P_M\theta - 3(1-\theta P_M)}{16P_M\theta + 3(1-\theta P_M)}}{16\theta - 13\theta\frac{16P_M\theta - 3(1-\theta P_M)}{16P_M\theta + 3(1-\theta P_M)}}$$

$$\Rightarrow \quad P_M[(16)(13\theta^2P_M) + (16)(3\theta) - (13)(19\theta^2P_M) + (13)(3\theta)]$$

$$= 64\theta P_M - 12\theta P_M - 16\theta P_M - 3\theta P_M + 12 + 3$$

$$\Rightarrow \quad -39\theta^2P_M^2 + 54\theta P_M - 15 = 0$$

$$\Rightarrow \quad P_M = \frac{54 + 24}{78\theta} = \frac{1}{\theta} \qquad (33)$$

or

$$P_M = \frac{-54 + 24}{-78\theta} = \frac{5}{13\theta}. \quad (34)$$

Recall that the manufacturer has the first mover's advantage. As such he will prefer a profit margin that will lead to higher payoff. Now observe that for all values of θ (33) is greater than (34). Thus there is the temptation for the manufacturer to take $P_M = \frac{1}{\theta}$ as the best option. But using this in (13) and (12) gives

$$P_R = P_D = P_M = \frac{1}{\theta}. \qquad (35)$$

The implication of using (35) in (5), (6) and (7) is that $P_{Prf} = D_{Prf} = M_{Prf} = 0$.

This makes no sense! On the other hand adopting (34) we have that (13) becomes

$$P_D = \frac{1 + 3\theta\frac{5}{13\theta}}{4\theta} = \frac{7}{13\theta},$$

and (12) becomes

$$P_R = \frac{1 + \theta\frac{7}{13\theta}}{2\theta} = \frac{10}{13\theta}.$$

Using (15), (34), (26) and (27) in (11) we have

$$A_{(\beta>0)}=\frac{\alpha^2\left(1-\theta\frac{10}{13\theta}\right)^2\left(\frac{10}{13\theta}-\frac{7}{13\theta}\right)^2}{2^2\left(1-\frac{16\left(\frac{5}{13\theta}\right)\theta-\left(1-\theta\frac{5}{13\theta}\right)}{16\left(\frac{5}{13\theta}\right)\theta+\left(1-\theta\frac{5}{13\theta}\right)}\right)^2}$$
$$=\left(\frac{3\alpha}{52\theta}\right)^2.$$

Using (26), (27) and (28) in (5) we have

$$R_{Prf}=\left(\frac{3}{13}\right)\left(\frac{3}{13\theta}\right)\alpha\left(\frac{\alpha}{4}\right)\left(\frac{3}{13\theta}\right)-\left(1-\frac{56}{104}\right)\frac{\alpha^2}{\theta^2}\left(\frac{3}{13\theta}\right)^2$$
$$=\left(\frac{3}{26}\right)^3\left(\frac{\alpha}{\theta}\right)^2.$$

Using (34), (26), (27) and (29) in (6) we have

$$D_{Prf}=\left(\frac{3}{13}\right)\left(\frac{7}{13\theta}-\frac{5}{13\theta}\right)\alpha\left(\frac{\alpha}{4}\right)\left(\frac{3}{13\theta}\right)$$
$$=\frac{9\alpha^2}{4394\theta^2}.$$

Using (34), (27) and (29) in (7) we have

$$M_{Prf}=\left(\frac{3}{13}\right)\left(\frac{5}{13\theta}\right)\alpha\left(\frac{\alpha}{4}\right)\left(\frac{3}{13\theta}\right)-\left(\frac{\alpha}{4}\right)^2\left(\frac{3}{13\theta}\right)^2\left(\frac{6}{13}\right)$$
$$=\frac{63\alpha^2}{175760\theta^2}.\quad\blacksquare$$

Numerical Illustration

Effect of subsidy on prices

For the purpose of illustration we let the advertising effectiveness $\alpha = 0.3$. Now,considering Fig. 1 we observe that when retail advertising is subsidised the players' pricesare higher. This suggests that the advertising expenditure (through subsidy) is compensated for through increase in prices. Unfortunately the distributor who is neither directly nor indirectly involved in advertising also increases his price. This obviously has a negative effect on the supply chain because the carry-over effect of his increased price extends to the retailer. Thus the retailer is bound to increase his price. This increase in retail price should naturally be justifiable. However, the increase in the distributor's price can have additional effect on the retail price. In general, this may affect sales response.

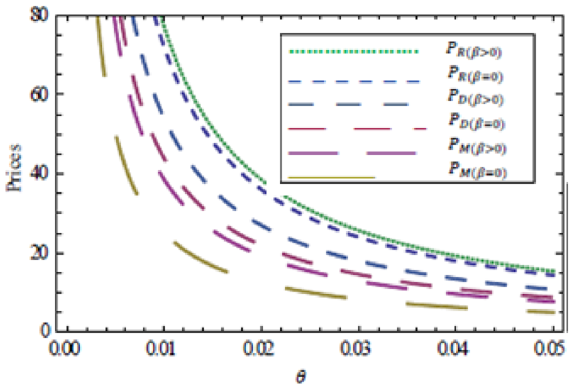

Fig. 1: Players' prices for subsidised and unsubsidised advertising effort

Comparison of advertising efforts

Figure 2 shows that with support from the manufacturer, the retailer is motivated towards advertising the manufacturer's product. Thus the subsidized advertising effort is higher than the unsubsidized advertising effort. It is pertinent to note that this new (increase in) effort is the result of the involvement of only two players: the retailer and the manufacturer. The distributor is neither directly nor indirectly involved.

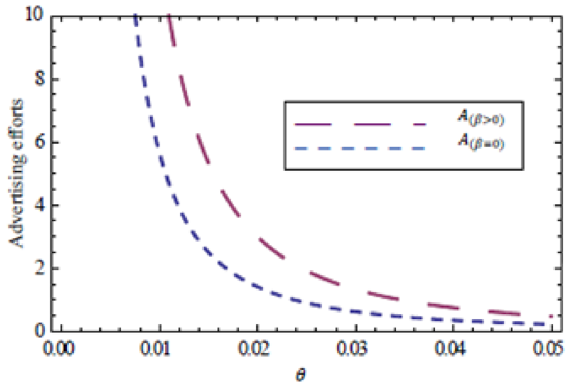

Fig. 2: Subsidised and unsubsidised advertising effort

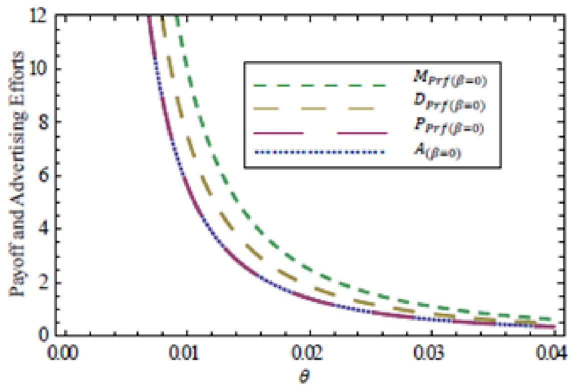

Fig. 3: Comparison of advertising effort and payoffs for unsubsidised advertising

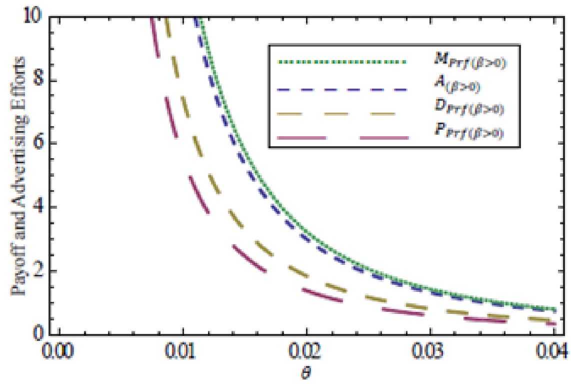

Fig. 4: Comparison of advertising effort and payoffs for subsidised advertising

Comparison of advertising efforts and payoffs

In Fig. 3 we have that without subsidy the retailer's advertising effort coincides with his payoff, thus implying that he has problem getting to a break-even point. This is coupled with the fact that the other players' (the manufacturer and distributor's) payoffs are larger. This situation is unpalatable because of the distributor who makes no advertising expenditure in the chain but has a higher payoff compared to the retailer who bears the burden of advertising. A far more disturbing situation can be found in Fig. 4 which gives the advertising effort and payoffs for subsidised retail advertising. Obviously, the advertising expenditure has increased from being equal to the unsubsidised retail payoff to being above the distributor's payoff. Unfortunately the retail payoff did not increase with his advertising expenditure. Thus we still have that $P_{Prf(\beta>0)} < D_{Prf(\beta>0)} < M_{Prf(\beta>0)}$. It therefore follows that the increase in advertising effort resulting from subsidy benefits the distributor more than it benefits the retailer.

Effect of advertising effort on payoffs

Looking at Fig. 5 we have that the unsubsidized retail payoff increases more rapidly with advertising than the subsidized retail payoff. Added to this is that it also peaks with a much lower effort compared to the subsidized, and this maximum value is higher than the subsidized maximum value. After these optimal (maximum) values, diminishing returns sets in with any additional effort. A similar picture can be seen in Fig. 6 where the subsidized manufacturer's payoff increases and gets to a maximum, then diminishing returns sets in. This is not the case with unsubsidized manufacturer's payoff which increases continuously with advertising, which is also the case with Fig. 7 where both the subsidized and unsubsidized distributor's payoff increases continuously with advertising. Thus the diminishing returns in Figs. 5 and 6 are as a result of the involvement of the retailer and manufacturer in advertising directly and indirectly, respectively.

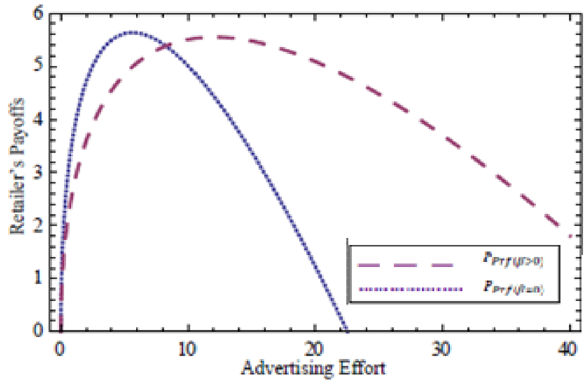

Fig. 5: Effect of advertising effort on subsidised and unsubsidised retailer's payoffs

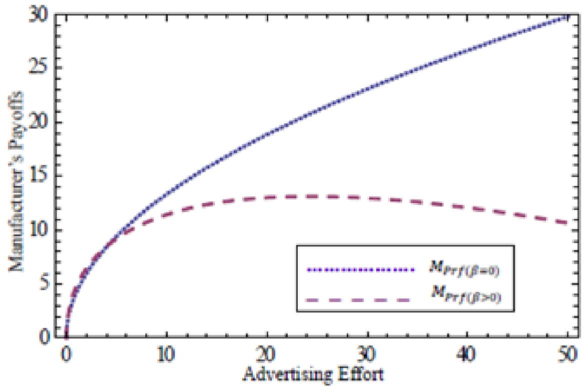

Fig. 6: Effect of advertising effort on subsidised and unsubsidised manufacturer's

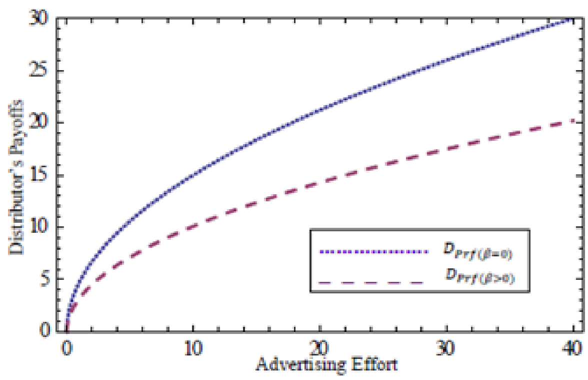

Fig. 7: Effect of advertising effort on subsidised and unsubsidised distributor's payoffs

Obviously, the noninvolvement of the distributor in advertising makes his payoff to continuously increase with the retailer and manufacturer's advertising involvements (that is without experiencing diminishing returns). Also, the noninvolvement of the manufacturer when there is no subsidy makes his payoff to continue to grow and become unbounded. Clearly, while the distributor who virtually contributes nothing to the supply chain has an ever-increasing payoff, the retailer and manufacturer experience diminishing returns. The retailer bears the burden of direct interaction with the consumer and advertising spending, while the manufacturer bears the burden of production and indirect advertising spending. This implies that such a distributor in such a supply chain setting – with three members – may be seen as a gold digger. As such for him to be incorporated into the chain he must be directly or indirectly involved in advertising.

Concluding Remarks

This work identified the optimal pricing and cooperative advertising strategies in a channel consisting of a manufacturer, a distributor and a retailer. It used three game-theoretic models to show that all the players must necessarily be either directly or indirectly involved in advertising. Particularly, it illustrated that irrespective of the amount of subsidy provided by the manufacturer to the retailer, the retailer is bound to be shortchanged if the distributor is not in any way involved in advertising.

This paper has added to the scanty game theory literature on cooperative advertising by using price, advertising effort and subsidy. However, it has some limitations. First, we assumed a trilateral monopoly static model. An extension can consider a situation where there is competition between two or all-three channel members. This may flavour the work with better insight. Secondly, we employed the linear demand-price function. If a different demand-price function is used, perhaps a significantly different outcome may be observed. This may lead to different managerial implications. In addition, different price, demand and cooperative advertising functions may be employed instead of the multiplicative form used.

Conflict of Interest

Author declares that there is no conflict of interest reported on this work.

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